Ministry of Education and Science of the Russian Federation
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Control Systems and Technology Department

Report for Laboratory No. 1

Simulation of Linear Control Systems Using Functions from the Control System Toolbox
Analysis of Characteristics for Direct Current Motor

Course: Mathematical Modeling and Simulation

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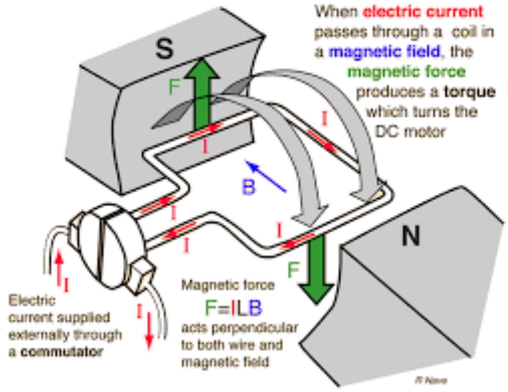
Introduction

Lab 1 is about simulation, analysis, and optimization of a DC motor. A motor is selected from an online catalog, providing required design constant values. Using these design constants, various plots and critical values are calculated.

Finally, using these plots and critical values, a summary of the motor characteristics and information for further optimization is produced.



The analysis software for this lab is Matlab, specifically the "Control System Toolbox". It will be used for producing Nyquist, Bode, step, and impulse charts, which produce important result data such as percent overshoot and settling time. All code used for this analysis can be seen in Appendix 1.

Background

Before starting with the analysis in Matlab, it is necessary obtain the transfer function of the DC motor. We start with the electrical representation of the DC motor, which is represented by a resistance part and an inductance part. In this case the relation between the input and output will be the angular motor speed and the voltage.

Transfer function of a DC motor:

$$H(s) = \frac{output(s)}{input(s)} = \frac{w(s)}{v(s)}$$

<u>Motor Constants</u>

Ra = Electrical resistance

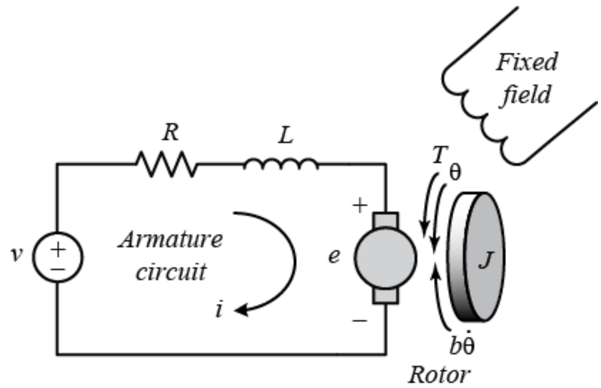
L = Electrical inductance

K_e = Electromotive force constant

K_m = Motor torque constant

J = Moment of inertia of the rotor

W_{DCM} = Angular motor speed



Relation between torque (T) and current (I):

$$T_m = K_m bI$$

Transfer function of torque equation:

$$\frac{T_m}{I} = K_m b$$

Relation between voltage and current mesh:

$$V_a = V_R + V_L + V_E = (R_a I) + L_a\left(\frac{dI}{dt}\right) + V_B$$

$V_E = K_e b \times W_{DCM}$ = Back EMF Voltage

The la place transform of this voltage equation:

$$V_a(s) - \left(K_b \times W_{DCM}(s)\right) = (R_a + L_a)s \times I(s)$$

The rotational motion of internal load:

$$\sum M = T_m - (K_e \times W_{DCM}) = J\dot{w_{DCM}}$$

Transfer function:

$$\frac{W_{DCM}(s)}{T_m(s)} = \frac{{}^{1}\!/_{J}}{s + {}^{K_e}\!/_{J}}$$

Replacing values and dividing by $\frac{1}{s}$ we obtain the analytical transfer function for a DC motor's angular speed is shown below. Using this transfer function, plots are produced and utilized in Matlab.

$$H(s) = \frac{\dfrac{{}^{1}\!/_{J}}{s + {}^{K_e}\!/_{J}}}{(R_a + L_a)s \times i + Kb * \dfrac{{}^{1}\!/_{J}}{s + {}^{K_e}\!/_{J}}}$$

$$H(s) = \frac{\dfrac{1}{Ke}}{\left(\dfrac{J * R_a}{K_e * K_m}\right)\left(\dfrac{L}{R_a}\right)s^2 + \dfrac{J * R_a}{K_e * K_m}s + 1}$$

For further clarity, the transfer function is simplified into defined variables.

<u>Defined Variables</u>

$$T_m = \frac{JR_a}{K_e K_m}$$

$$T_a = \frac{L}{R_a}$$

$$K_m = K_e$$

<u>Transfer Function</u>

$$W_{DCM}(s) = \frac{{}^{1}\!/_{K_e}}{T_m T_a s^2 + T_m s + 1}$$

<u>Plots</u>

Four different plots are utilized to analyze the motor design. Each plot provides important information about the effective usage of a desired motor.

1.) Nyquist – Stability analysis
2.) Bode – Magnitude and frequency analysis
3.) Step – Peak overshoot and settling time
4.) Impulse – Response magnitude and settling time

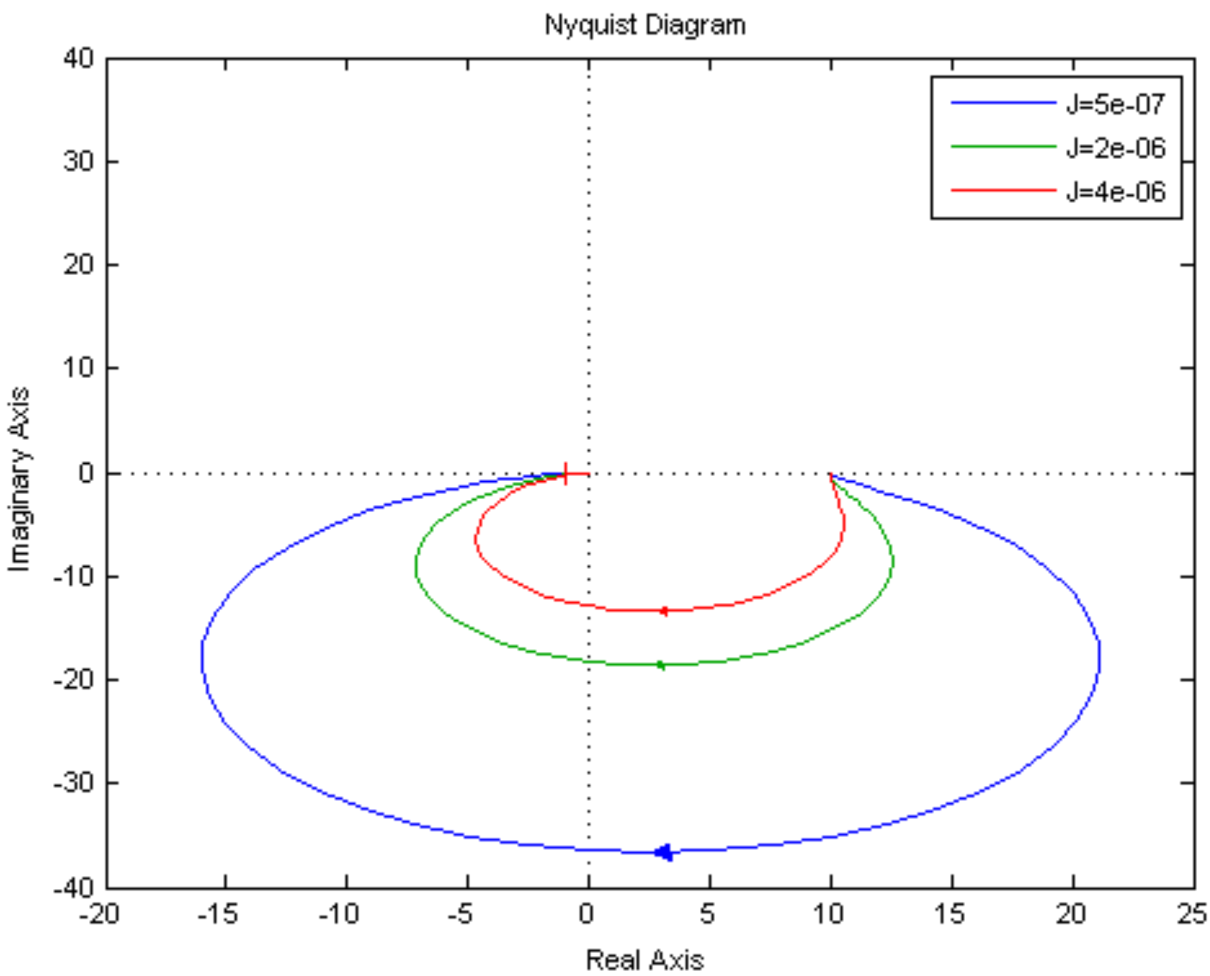
Analysis of Frequency Characteristics

A random motor was chosen with the following characteristics. This motor has been analyzed via the Nyquist, Bode, Step, and Impulse functions to determine an optimization point for the angular speed (W_{DCM}). Additionally, various inertia values (J) are tested to account for different objects attached to the rotor.

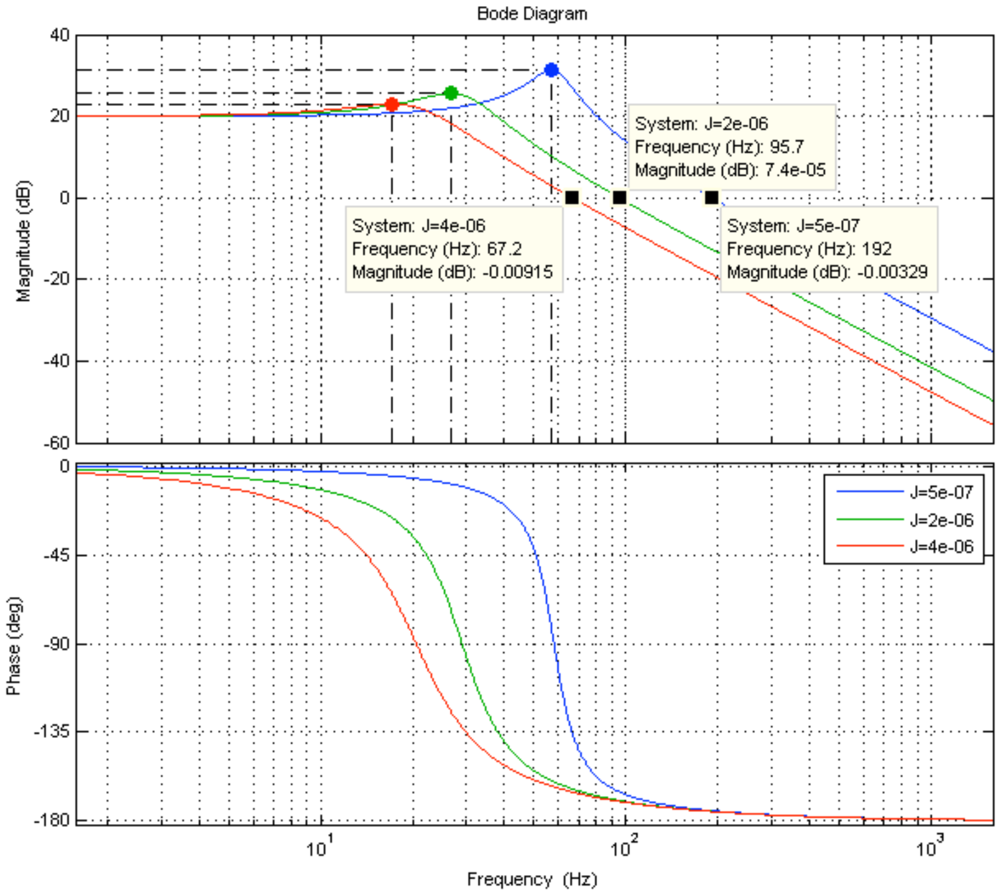
Ra	= 15	Ohms	= Electrical Resistance
L	= 0.15	Henrys	= Electrical Inductance
K_e	= 0.1	V/(rad/sec)	= Electromotive force constant
K_m	= K_e	Nm / P^2	= Motor torque constant
J	= (input)	Kg/m^2	= Moment of inertia of the rotor
W_{DCM}	= (output)	rad/sec	= Angular motor speed

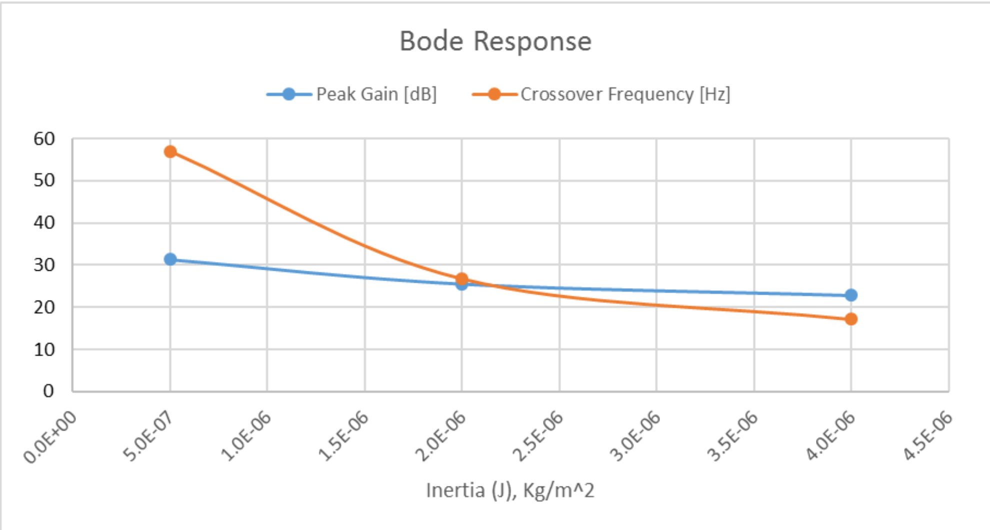
The inertia (J) was modified from 0.05E-6 to 0.40E-6 with 0.05E-6 intervals. This can be seen in the legend of each chart.

Nyquist Diagram



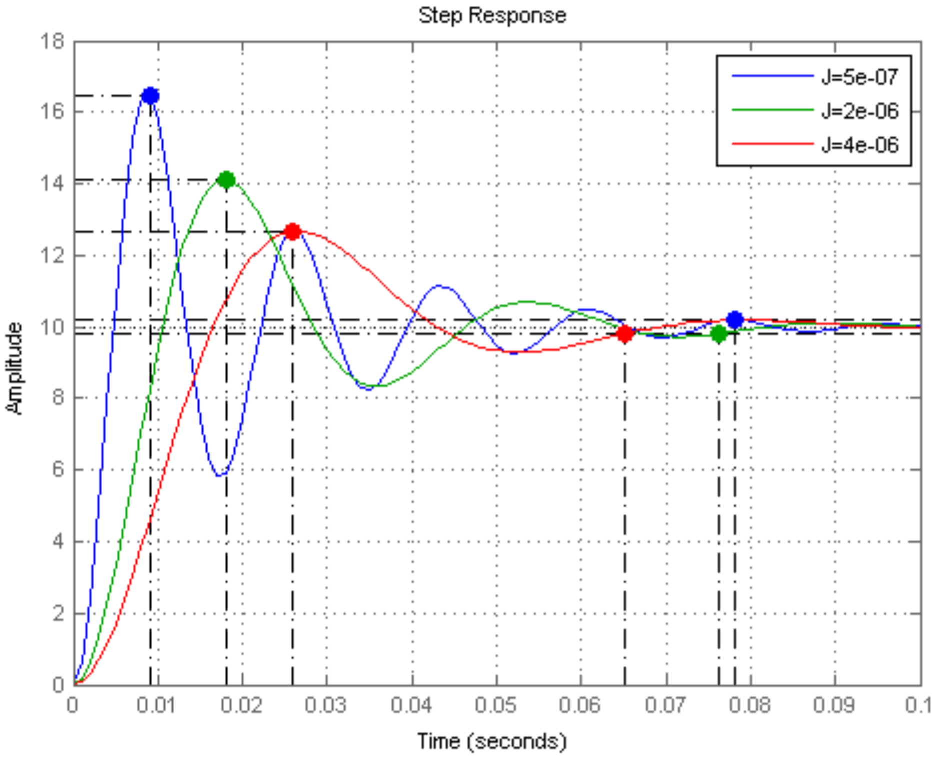
Bode Diagram

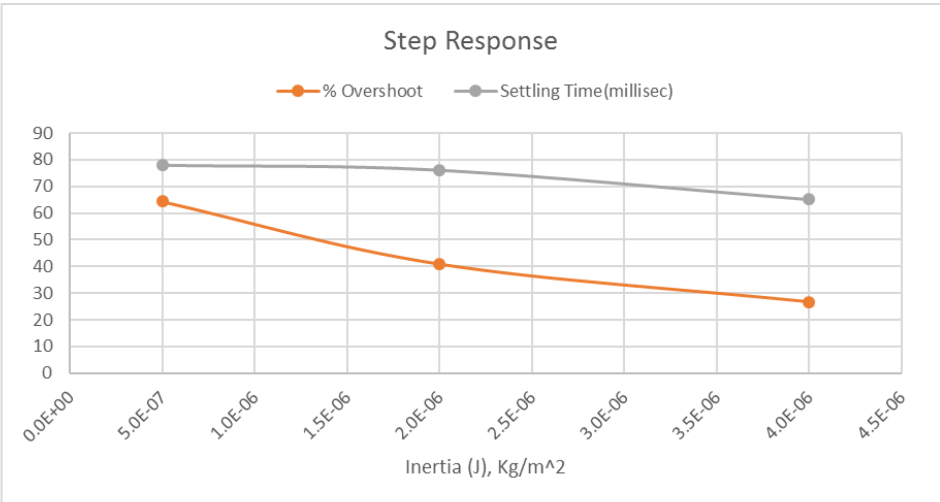




Inertia (J)	Peak Gain [dB]	Crossover Frequency [Hz]
5.0E-07	31.3	192
2.0E-06	25.6	95.7
4.0E-06	22.9	67.2

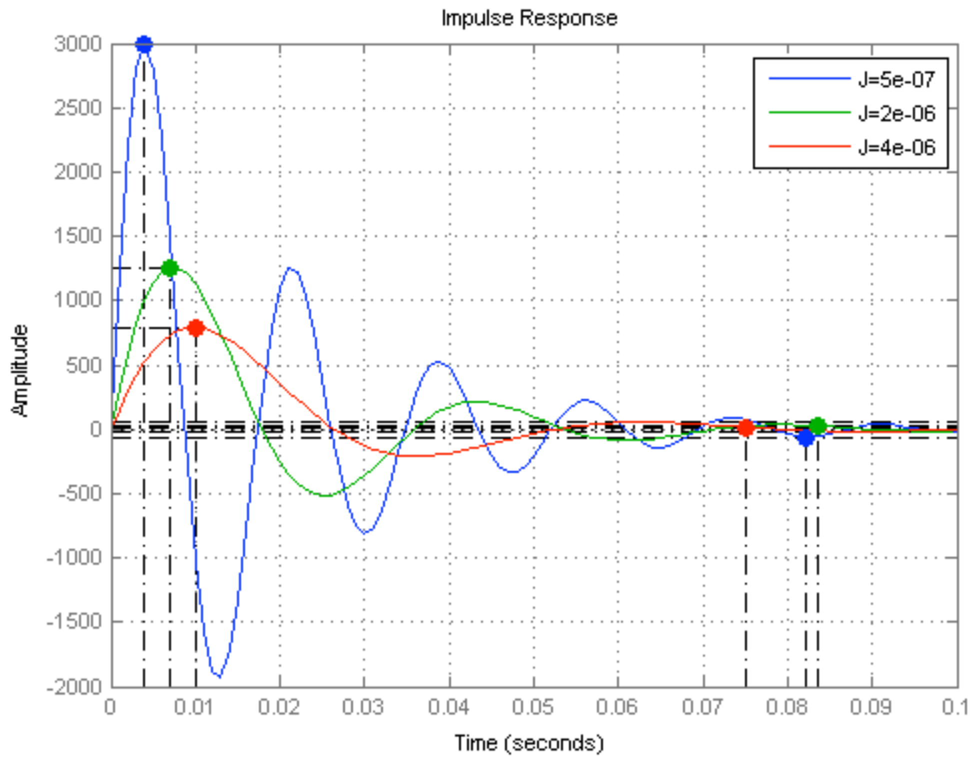
Step Response

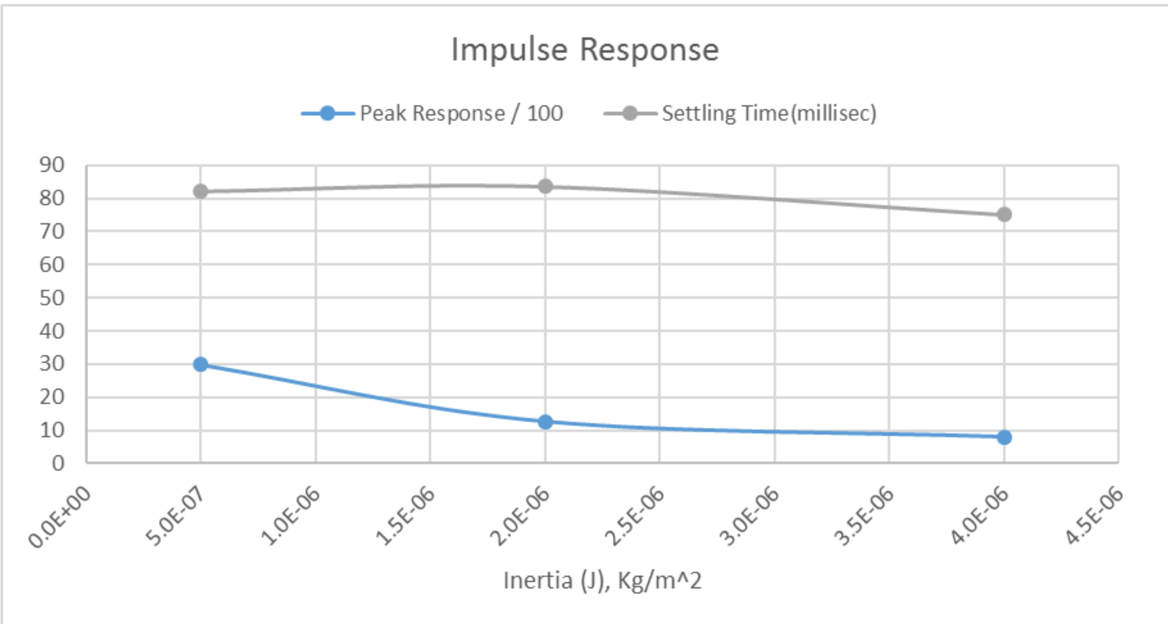




Inertia (J)	**% Overshoot**	**Settling Time [sec]**	**Settling Time [millisec]**
5.0E-07	64.4	0.0780	78
2.0E-06	40.9	0.0761	76.1
4.0E-06	26.7	0.0651	65.1

Impulse Response





Inertia (J)	Peak Response	Peak Response / 100	Settling Time [sec]	Settling Time [millisec]
5.0E-07	2.99E+03	29.9	0.0821	82.1
2.0E-06	1.26E+03	12.6	0.0835	83.5
4.0E-06	789	7.89	0.0750	75

Conclusion

Stability analysis – the Nyquist plot shows the following trends.

1.) The motor is stable for all frequencies.
2.) The motor is stable regardless of inertia.

Bode Plot – Gain and crossover frequency.

1.) Peak gain decreases from 31.3dB to 22.9dB as inertia increases.
2.) The crossover frequency decreases from 192 to 67.2 as inertia increases.

Step Chart – Overshoot and Settling Time

1.) The overshoot decreases by 37.7% as the inertia increases.
2.) The settling time is not very affected by the inertia.

Impulse Chart – Response Magnitude and Settling Time

1.) The peak response drops from ~3000 to 789 as inertia increases.
2.) The settling time is not very affected by the inertia.

Appendix 1 – Matlab Code

```
%Lab1(Simulation of Linear Control Systems using Functions From Control
System Toolbox)
%Analysis of Characteristics for Direct Current Motor
clear all; close all; clc

%% Motor Properties
Um_max = 30;     % Max applied voltage           Units: volts
Wmax = 300;      % Max velocity                  Units: rad/sec
Ra=15;           % Electrical resistance         Units: ohms
La= 0.15;        % Electrical inductance         Units: henry
J=0.12E-5;       % Inertia                       Units: Kg*m^2

Ta = La/Ra;      % Inductance Resistance Ratio
Ke=Um_max/Wmax;  % Electromotive force constant  Units: volts / (rad/sec)
Km=Ke;           % Torque constant               Units: Nm / (Work)^2

%% Create transfer functions for each inertia
i = 0;
for J = [5.0E-7, 2.0E-6, 4.0E-6]
    i = i + 1;
    %Store transfer functions
    Tm=(J*Ra)/(Ke*Km);
    theTFs(i) = tf(1/Ke, [Tm*Ta Tm 1]);

    %Store legend entries
    theLegend(i) = {['J=' num2str(J)]};
end
L = i; % Number of series in charts.

%% Nyquist
figure(1); hold;
P = nyquistoptions; P.FreqUnits = 'Hz';
for i = 1:L
    nyquistplot(theTFs(i), P);
end
legend(theLegend);

%% Bode
figure(2); hold;
P = bodeoptions; P.FreqUnits = 'Hz';
for i = 1:L
    bodeplot(theTFs(i), {10,10000}, P); grid on
end
legend(theLegend);

%% Step
figure(3); hold;
for i = 1:L
     step(theTFs(i), 0:0.001:0.15); grid on
end
legend(theLegend);

%% Impulse
figure(4); hold;
for i = 1:L
impulse(theTFs(i), 0:0.001:0.15); grid on
end
legend(theLegend);
```