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Report 5

Multi-Criteria Decisions with Uncertainty
Discipline: Modern Problems of Informatics and Computer Science
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Problem – SciTools Bidding

Problem Description

SciTools Incorporated, a company that specializes in scientific instruments, has been invited to make a bid on a government contract. The contract calls for a specific number of these instruments to be delivered during the coming year. The bids must be sealed (i.e. no company knows what the other companies are bidding). The lowest bid wins the contract.

SciTools estimates that it will cost $5000 to prepare a bid and $95000 to supply the instruments if it wins the contract. On the basis of past contracts, SciTools believes that the possible low bids from competition (if any), and their probabilities are those shown in the following table. In addition, SciTools believes there is a 30% chance of no competing bids.

What should SciTools bid to maximize its EMV? Task: Develop a decision model that finds the EMV for various bidding strategies and indicates the best strategy.

Solution

An EMV (Expected Monetary Value) approach is utilized, in combination with the following table of competitor predictions. The company will take a 3-tier approach to bidding that includes a conservative, medium, and optimistic approach. Three types of competitors are also created, each with their forms of conservative, medium, and optimistic bids. See below table.

SciTools Possible Bids(x $100)

Conservative	$100
Medium	$150
Optimistic	$200

Competitor Possible Bids (x $1000)

The below table represents the expected bidding ranges of three different competitors styles.

	Competitor 1 (Conservative)	Competitor 2 (Medium)	Competitor 3 (Optimistic)
Low	$115	$145	$175
Medium	$145	$190	$250
High	$175	$250	$300

Payoff Table

The bid options and competitor tables are combined into a possible payoffs table. Costs are show as negative (-) and rewards are shown as positive (+).

		Possible Competitor Bids								
		Competitor 1 - Cons.			Competitor 2 - Med.			Competitor 3 - Opt.		
		c1Low	c1Med	c1High	c2Low	c2Med	c2High	c3Low	c3Med	c3High
Strategies	Bid Amount	115	145	175	145	190	250	175	250	300
D1 - Cons.	110	10	10	10	10	10	10	10	10	10
D2 - Med.	150	-5	-5	50	-5	50	50	50	50	50
D3 - Opt.	200	-5	-5	-5	-5	-5	100	-5	100	100

Expected Monetary Value (EMV) Table

Finally the possibility of each bid is incorporated and the EMV is calculated. The belief of the company having a 30% chance of no competition is reflected in the low bids of each company and the 0.1 (10%) probability associated with them.

	Possible Competitor Bids									
	Competitor 1 - Pes.			Competitor 2 - Med.			Competitor 3 - Opt.			
	c1Low	c1Med	c1High	c2Low	c2Med	c2High	c3Low	c3Med	c3High	
Probability	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	
Strategies										Total EMV
D1 - Cons.	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	**9.00**
D2 - Med.	-0.5	-0.5	5.0	-0.5	5.0	5.0	5.0	5.0	5.0	**28.50**
D3 - Opt.	-0.5	-0.5	-0.5	-0.5	-0.5	10.0	-0.5	10.0	10.0	**27.00**

Conclusion

The above EMV table shows that the highest EMV result is with a **medium approach**. However, the optimistic approach has only a slightly lower EMV. Hence, it may be worth the risk to take the optimistic approach.

Problem – Student Rankings

Problem Description

A group of 8 students has been graded by various different experts. Each student is ranked in order. (1=best and 8=worst) Given a table of students and their rankings (see below), provide a method for the teacher to determine final rankings for the students.

Student ID	Exp. 1	Exp. 2	Exp. 3	Exp. 4	Exp. 5	Exp. 6	Exp. 7	Exp. 8	Exp. 9	Exp. 10
1	1	3	1	2	1	3	1	2	3	1
2	4	4	6	6	3	1	2	1	4	3
3	3	1	4	5	5	4	4	4	5	2
4	8	2	2	3	2	2	3	3	3	8
5	2	8	3	4	6	7	6	6	2	7
6	7	6	7	8	4	6	8	7	8	4
7	5	5	4	1	7	5	5	5	7	5
8	6	7	8	7	8	8	7	8	1	8

Procedure Explanation

Direct Sum Method

The ranking for each student for each expert's judgment is simply added. The student with the lowest value is the best student and the student with the highest value is the worst.

$$StudentScore_s = \sum_{e=1}^{10} rank_{se}$$

where "s" is the student ID and "e" is the expert ID.

The largest problem with this method is the clear, likely undesirable, weights of the ranking system. For example, let's compare students 1 and 4. If a student receives a ranking of 8, they are extremely penalized, possibly making it impossible for them to make up the mistake.

Student 1 has 5 rankings of 1, which is 5 points.
Student 4 has 2 rankings of 8, which is 16 points.

Weighted Percentage Method

A weight factor is included into the system to allow the teacher to decide the relative weight of earning a rank of 1,2,….,8. This allows students who received many slightly lower scores to outperform another student with only a couple better scores. The process is divided into the following steps.

1. **Count Rankings** - For each student, count how many of each ranking was received.

$$RankingCount_{sr} {+}{=} \sum_{e=1}^{\#\ Experts} \begin{cases} 1, & rank_e = r \\ 0, & rank_e <> r \end{cases}$$

   where "s" is the student ID, "r" is the possible rank value, and "e" is the expert ID.

2. **Convert Rankings to Percentages** - Count the total of all rankings, which should equal the number of students multiplied by the number of experts. Divide the rankings count by the total.

$$\% \, RankingCount_{sr} = \frac{RankCount_{sr}}{\sum_{r=1}^{\#\,Rakings} \sum_{s=1}^{\#\,Students} RankCount_{sr}}$$

where "s" is the student ID and "r" is the possible rank value.

3. **Application of Weights** – The teacher applies a chosen set of weights, which give preference to different ranking values. The weights must sum to 1.0. For example, a "1" may be given a weight of 0.20 and a rank of "2" may be given a weight of 0.19. This makes earning a rank of "1" only slightly better than a rank of "2".

$$Score_{sr} = W_r * \% \, RankingCount_{ir}$$

4. **Sum Scores** – The scores are added for each student. The maximum score is the best student, and the minimum score is the worst student.

$$Score_s = \sum_{r=1}^{\#Rankings} Score_{sr}$$

Solution

The previously discussed two methods were implemented in excel. Below are the results of each operation and a comparison of the final results from each method.

Direct Sum Method

The sum and final ranking of each student is shown on the right.

Student ID	**Exp. 1**	**Exp. 2**	**Exp. 3**	**Exp. 4**	**Exp. 5**	**Exp. 6**	**Exp. 7**	**Exp. 8**	**Exp. 9**	**Exp. 10**	**Sum**	**Rank**
1	1	3	1	2	1	3	1	2	3	1	**18**	**1**
2	4	4	6	6	3	1	2	1	4	3	**34**	**2**
3	3	1	4	5	5	4	4	4	5	2	**37**	**4**
4	8	2	2	3	2	2	3	3	3	8	**36**	**3**
5	2	8	3	4	6	7	6	6	2	7	**51**	**6**
6	7	6	7	8	4	6	8	7	8	4	**65**	**7**
7	5	5	4	1	7	5	5	5	7	5	**49**	**5**
8	6	7	8	7	8	8	7	8	1	8	**68**	**8**

Weighted Percentage Method

Ranking Counts

The below table is a count of each ranking possibility for each student.

Student ID	**Count 1s**	**Count 2s**	**Count 3s**	**Count 4s**	**Count 5s**	**Count 6s**	**Count 7s**	**Count 8s**
1	5	2	3	0	0	0	0	0
2	2	1	2	3	0	2	0	0
3	1	1	1	4	3	0	0	0
4	0	4	4	0	0	0	0	2
5	0	2	1	1	0	3	2	1
6	0	0	0	2	0	2	3	3
7	1	0	0	1	6	0	2	0
8	1	0	0	0	0	1	3	5

Conversion to Percentages

There sum of all values in the previous table is 80. Hence each value is divided by this to normalized its influence.

Student ID	Count 1s	Count 2s	Count 3s	Count 4s	Count 5s	Count 6s	Count 7s	Count 8s
1	0.0625	0.025	0.0375	0	0	0	0	0
2	0.025	0.0125	0.025	0.0375	0	0.025	0	0
3	0.0125	0.0125	0.0125	0.05	0.0375	0	0	0
4	0	0.05	0.05	0	0	0	0	0.025
5	0	0.025	0.0125	0.0125	0	0.0375	0.025	0.0125
6	0	0	0	0.025	0	0.025	0.0375	0.0375
7	0.0125	0	0	0.0125	0.075	0	0.025	0
8	0.0125	0	0	0	0	0.0125	0.0375	0.0625

Application of Weights and Final Rankings

In the given example the following weights were applied to each ranking category, allowing rankings to be more/less similar. The sum and final rankings are shown on the right.

Weights:	0.20	0.19	0.18	0.14	0.10	0.08	0.06	0.05		
Student	**Count 1s**	**Count 2s**	**Count 3s**	**Count 4s**	**Count 5s**	**Count 6s**	**Count 7s**	**Count 8s**	**Sum**	**Rank**
1	1.3	0.5	0.7	0.0	0.0	0.0	0.0	0.0	**2.4**	**1**
2	0.5	0.2	0.5	0.5	0.0	0.2	0.0	0.0	**1.9**	**3**
3	0.3	0.2	0.2	0.7	0.4	0.0	0.0	0.0	**1.8**	**4**
4	0.0	1.0	0.9	0.0	0.0	0.0	0.0	0.1	**2.0**	**2**
5	0.0	0.5	0.2	0.2	0.0	0.3	0.2	0.1	**1.4**	**5**
6	0.0	0.0	0.0	0.4	0.0	0.2	0.2	0.2	**1.0**	**7**
7	0.3	0.0	0.0	0.2	0.8	0.0	0.2	0.0	**1.3**	**6**
8	0.3	0.0	0.0	0.0	0.0	0.1	0.2	0.3	**0.9**	**8**

Note: The results matrix has been multiplied by a factor of 100 to make it easier to read.

Conclusion

Below is a comparison of the rankings from both methods. The highlighted lines show different results between the methods. Both methods agreed on the worst and best students. However, they different on the students in the middle.

Student ID	Direct Sum	Weighted %
1	1	1
2	2	3
3	4	4
4	3	2
5	6	5
6	7	7
7	5	6
8	8	8